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RESEARCH ARTICLE

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Optimal amounts of water and nitrogen applied to sugar beet when crop price depends on its sugar content

Ali Shabani (Shabani, A)[1] and Ali Reza Sepaskhah (Sepaskhah, AR)[2,3]

[1]*Fasa University, Water Science and Engineering Dept., Daneshjou blvd., Fasa, Fars Province, I.R. of Iran.* [2]*Shiraz University, Irrigation Dept., Shiraz, Badjgah, Fars Province, I.R. of Iran.* [3]*Shiraz University, Drought Research Center, Shiraz, Badjgah, Fars Province, I.R. of Iran.*

Abstract

Aim of study: To derive mathematical formulas to determine the optimum amounts of applied water and N at variable crop prices and rainfall conditions for sugar beet.

Area of study: Karaj Research Center, Alborz Province, Iran.

Material and methods: At first, mathematical formulas were derived to determine optimum applied water and nitrogen for sugar beet under rainfall occurrence, land limited (in cases that arable land area is limited and the farmer can not put more land area under irrigation) and water limited conditions when crop price depends on sugar content. Second, this theory was applied to analyze the relevant experimental data. The experiment was a split-plot design with irrigation treatments as the main plots (40%, 80%, 120% and 160% of evaporation from the surface of class A evaporation pan) and N fertilizer rates (0, 90, 180 and 270 kg N/ha) as subplots.

Main results: Under land and water limiting conditions, deficit irrigation of 27% and 48% led to 6.4% and 25.4% decrease in yield and 21.4% and 96.2% increase in total net income, respectively, compared with full irrigation. Under water limiting conditions, cultivated land area increased by 93.7, 108 and 128% for 0, 60 and 120 mm rainfall, respectively. Under land limiting conditions, amounts of optimum irrigation water were 12381.2, 11781.2 and 11181.2 m^3/ha, for 0, 60 and 120 mm rainfalls, respectively. The corresponding values for N were 262.5 kg/ha in all three rainfall quantities. Besides, under water limiting conditions, optimum amounts of irrigation water were 8708.1, 7828.8 and 6882.1 m^3/ha for 0, 60 and 120 mm rainfalls, respectively. The corresponding values for N were 301.1, 299.5 and 295.5 kg/ha, respectively. Optimum amounts of irrigation water and N decreased by increase in rainfall amount.

Research highlights: Under limited irrigation water conditions, if the rainfall, residual N, water cost and base crop price increases, the value of optimum applied water should be decreased.

Additional keywords: water limiting conditions; land limiting conditions; water management.

Abbreviations used: A (irrigated area); I_f (total net income from all irrigated area); i_l (net income per unit area); N_l (optimum level of N fertilizer under limited land conditions); N_m (amount of applied N which results in maximum yield); N_r (soil residual mineral N); N_w (optimum level of N fertilizer under limited water conditions); P_c (crop price); P_{c16} (base price of sugar beet); P_n (N fertilizer cost); P_w (water price); R (rainfall); SC (sugar concentration); W (applied water); W_l (optimum level of water under limited land conditions); W_m (applied water which results in maximum yield); W_w (optimum level of water under limited water conditions).

Authors' contributions: Both authors participated in the conception and design of the research, analysis or interpretation of data, and writing of the paper.

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Correspondence should be addressed to Ali Shabani: shabani8ali@gmail.com

Introduction

Application of extra nitrogen (N) fertilizers and irrigation water result in water resources pollution (Rahmati *et al.*, 2015; Barzegari *et al.*, 2017) and decrease in yield and farmer income in agriculture (Ehteshami & Biglarijoo, 2014; Rahmati *et al.*, 2015). For example, in the north of Iran nitrate concentration in groundwater in citrus orchard areas is considerably lower than in rice fields because of less irrigation and less fertilizer use in citrus orchards (Ehteshami & Biglarijoo, 2014). On the other hand, inadequate applications of N fertilizers and irrigation water result in decrease in crop yield and income. Therefore, irrigation and N fertilizer rates and its application in different plant growth phases influence the amounts of N leaching (Barton & Colmer, 2006). English *et al.* (1990) and English & Raja (1996) performed an economic analysis to determine the optimum applied water, and Zand-Parsa & Sepaskhah (2001) and Sepaskhah *et al.* (2006) determined the optimum applied water and N. They showed that the amount of applied water and N which resulted in maximum net income per unit of applied water or land, is lower than the level of irrigation water and N that maximises the crop yield. Therefore, determining the optimum applied N and irrigation water, based on economic analysis, is very important to decrease N leaching, groundwater contamination and economic loss. In all the above-mentioned studies, the analyses were conducted assuming a constant crop price. However, for sugar beet (*Beta vulgaris* L. subsp. *vulgaris* var. *altissima*), crop price is dependent on sugar content. In other words, sugar concentration of sugar beet is measured in sugar refining factories, then crop price is determined based on its value. The higher the percentage of sugar content, the higher the price of sugar beet (Jalilian *et al.*, 2001; Loel *et al.*, 2014; Khorramian & Hosseinpour, 2016).

Irrigation water and N fertilizers affect the sugar content of sugar beet (Hoffmann, 2010). Water stress is leading to proline, glycine betaine, glucose and sucrose accumulation and ultimately on sugar concentration in the sugar beet (Winter, 1988; Rinaldi & Vonella, 2006; Choluj *et al.*, 2008). Increasing N levels decreases sugar content of sugar beet (Akeson *et al.*, 1979; Monreal *et al.*, 2007). Sadeghi-Shoae *et al.* (2015) indicated that, applying increasing amounts of N fertilization, sugar concentration increased if N had low concentration, and decreased if the concentration was high. Carter (1982) showed that before root yield reached its maximum level, increase in applied N fertilizer to a certain and optimum level resulted in increase in sugar beet root yield, although, its sugar concentration decreased by increasing N.

Sepaskhah & Akbari (2005) and Sepaskhah *et al.* (2006) considered seasonal rainfall in an economic analysis to determine the optimum applied water for cotton and wheat, respectively. They modified an equation to determine the optimum irrigation water for different years with different situations of seasonal rainfall. These investigators showed that the optimum amount of irrigation water for different years increased as the seasonal rainfall decreased. Despite there is uncertainty and risk related to estimates of optimum applied water, farmers will accept some degree of risk if there is a potential economic gain (Sepaskhah & Akbari, 2005).

Optimization problems can be solved by means of mathematical programming or heuristic methods. Mathematical solutions to the problems are more understandable, analyzable and result in exact answers. However, heuristic methods do not necessarily lead to exact answers, and researchers should be consent to obtain the answers with an acceptable accuracy (Burke *et al.*, 2003). These methods must be applied in cases where the mathematical solutions cannot be used or solved (Rodríguez *et al.*, 2018). Furthermore, heuristics methods are black-box type (Muñoz *et al.*, 2015) that may result in a final answer that may be incorrect (Kaveh & Talatahari, 2010). The heuristic methods present just one answer, and not necessarily the optimal one, while the case may have more than one answer (Feng *et al.*, 1997; Mulder, 2018). It is easier to understand issues based on our mathematical methods. Sensitivity analysis in mathematical methods is much easier and more comprehensible, but in heuristic and non-mathematical methods, the software must be run several times with different inputs, what is time-consuming (Yoo & Kim, 2014).

In this study, mathematical formulas were derived to determine the optimum amounts of applied water and N at variable crop prices and rainfall conditions for sugar beet under land and water limiting conditions. This theory was applied for sugar beet data obtained in Alborz Province of Iran (Karimi & Naderi, 2008).

Material and methods

Mathematical formulation with variable crop prices

The mathematical formulation was presented by Zand-Parsa & Sepaskhah (2001) to determine the farm net income in case that the crop yield and price are a function of applied water and N as follows:

$$I_f(W+R, N+Nr) = A.\, i_l(W+R, N+Nr) \quad (1)$$

$$i_l(W+R, N+Nr) = = [p_c(W+R, N+Nr) * y(W+R, N+Nr) - c(W, N)] \quad (2)$$

where A is the irrigated area (ha), W is the applied water (m^3/ha), R is the rainfall (m^3/ha), N is the N application rate (kg/ha), Nr is the soil residual mineral N (kg/ha), y (W+R, N+Nr) is the crop yield (kg/ha), c (W, N) is the production costs (Rls[1]/ha), P_c (W+R, N+Nr) is the crop

[1]Each US$ was equivalent to 8280 Iranian Rials at the time of carrying out the experiment.

price (Rls/kg), i_l(W+R, N+Nr) is the net income per unit area (Rls/ha), and I_f (W+R, N+Nr) is the total net income (Rls) from all irrigated area. Contrary to W and N that are management parameters and can be selected, R and Nr are not easily available under field conditions; however, R can be predicted. Although the values of R and Nr affect the crop yield and price, they do not influence the crop production costs.

The irrigated area (A) can be a function of used water and is not dependent on N fertilizer because of no limitation for N use. The irrigated area can be determined as follows:

$$A = \frac{W_T}{W} \quad (3)$$

where W_T is the total available water supply (m^3).

Under land limited conditions (when the arable land area is limited), the farmer put all available arable land under irrigation and cannot increase the land area. Therefore, net income per unit area gets a maximum value since the partial derivative of farm net income equation [Eqn (1)] with respect to W and N was set to zero as follows:

$$\frac{\partial I_f(W+R,N+Nr)}{\partial W} = A.\frac{\partial i_l(W+R,N+Nr)}{\partial W} + i_l(W+R,N+Nr)\frac{\partial A}{\partial W} = 0 \quad (4a)$$

$$\frac{\partial I_f(W+R,N+Nr)}{\partial N} = A.\frac{\partial i_l(W+R,N+Nr)}{\partial n} + i_l(W+R,N+Nr)\frac{\partial A}{\partial N} = 0 \quad (4b)$$

Under land limiting conditions, all available land is put under irrigation and A is not dependent on applied water and N. In this condition, the derivative of A is zero. Therefore, the optimum levels of water and N fertilizer under limited land conditions, *i.e.*, W_l and N_l can be determined as follows:

$$\frac{\partial i_l(W+R,N+Nr)}{\partial W} = 0 \quad (5a)$$

$$\frac{\partial i_l(W+R,N+Nr)}{\partial N} = 0 \quad (5b)$$

Therefore, the partial derivatives of Eqn (2) can be written as follows:

$$\frac{\partial\left[\left[p_c(W_l+R,N_l+Nr)*y(W_l+R,N_l+Nr)-c(W_l,N_l)\right]\right]}{\partial W} = 0 \quad (6a)$$

$$\frac{\partial\left[\left[p_c(W_l+R,N_l+Nr)*y(W_l+R,N_l+Nr)-c(W_l,N_l)\right]\right]}{\partial N} = 0 \quad (6b)$$

Therefore, under limited land conditions, optimum applied water and N can be determined by solving:

$$p_c(W_l+R,N_l+Nr)*\frac{\partial y(W_l+R,N_l+Nr)}{\partial W} + \frac{\partial p_c(W_l+R,N_l+Nr)}{\partial W}*\mathrm{y}(W_l+R,N_l+Nr) - \frac{\partial c(W_l,N_l)}{\partial W} = 0 \quad (7a)$$

$$p_c(W_l+R,N_l+Nr)*\frac{\partial y(W_l+R,N_l+Nr)}{\partial N} + \frac{\partial p_c(W_l+R,N_l+Nr)}{\partial N}*\mathrm{y}(W_l+R,N_l+Nr) - \frac{\partial c(W_l,nN_l)}{\partial N} = 0 \quad (7b)$$

Under water limiting conditions, the farmer may put more land area under irrigation to use all the water supply. Therefore, when the applied water and N are limited and non-limited, respectively, the partial derivative of A with respect to W and N can be written as follows:

$$\frac{\partial A}{\partial W} = -\frac{W_T}{W^2} \quad (8a)$$

$$\frac{\partial A}{\partial N} = 0 \quad (8b)$$

By substituting Eqns (2), (3), (8a) and (8b) in Eqns (4a) and (4b), the optimum amount of applied water and N under water limiting conditions, *i.e.* W_w and N_w, can be calculated as follows:

$$\frac{W_T}{W_w}\left[p_c(W_w+R,N_w+Nr)*\frac{\partial y(W_w+R,N_w+Nr)}{\partial W} + \frac{\partial p_c(W_w+R,N_w+Nr)}{\partial W}*\mathrm{y}(W_w+R,N_w+Nr) - \frac{\partial c(W_w,N_w)}{\partial W}\right] + \left[p_c(W_w+R,N_w+Nr)*y(W_w+R,N_w+Nr) - c(W_w,N_w)\right]\left[-\frac{W_T}{{W_w}^2}\right] = 0 \quad (9a)$$

$$\frac{W_T}{W_w}\left[p_c(W_w+R,N_w+Nr)*\frac{\partial y(W_w+R,N_w+Nr)}{\partial N} + \frac{\partial p_c(W_w+R,N_w+Nr)}{\partial N}*\mathrm{y}(W_w+R,N_w+Nr) - \frac{\partial c(W_w,N_w)}{\partial N}\right] + \left[\left[p_c(W_w+R,N_w+Nr)*y(W_w+R,N_w+Nr) - c(W_w,N_w)\right]\right]*0 = 0 \quad (9b)$$

By simplifying the Eqns (9a) and (9b)

$$W_w\left[p_c(W_w+R,N_w+Nr)*\frac{\partial y(W_w+R,N_w+Nr)}{\partial W} + \frac{\partial p_c(W_w+R,N_w+Nr)}{\partial W}*\mathrm{y}(W_w+R,N_w+Nr) - \frac{\partial c(W_w,N_w)}{\partial W}\right] = \left[p_c(W_w+R,N_w+Nr)*y(W_w+R,N_w+Nr) - c(W_w,N_w)\right] \quad (10a)$$

$$p_c(W_w+R,N_w+Nr)*\frac{\partial y(W_w+R,N_w+Nr)}{\partial N} + \frac{\partial p_c(W_w+R,N_w+Nr)}{\partial N}*\mathrm{y}(W_w+R,N_w+Nr) - \frac{\partial c(W_w,N_w)}{\partial N} = 0 \quad (10b)$$

and solving Eqns (10a) and (10b) for W and N will yield the optimum amount of applied water (W_w) and nitrogen (N_w) under limited water conditions and variable crop price.

The applied water and N amounts which result in maximum yield, *i.e.*, W_m and N_m, can be determined by taking the partial derivative of the crop yield function with respect to W and N and set those equal to zero as:

$$\frac{\partial y(W_m,N_m)}{\partial W} = 0 \quad (11a)$$

$$\frac{\partial y(W_m,N_m)}{\partial N} = 0 \quad (11b)$$

Since crop yield and price equations are nonlinear equations as a function of W and N, a nonlinear system of equations should be used for solving Eqns (7a), (7b), (10a), (10b) and (11a), (11b). In the current study, MATLAB software was used to solve this nonlinear system of equations.

Field experiment

The data used in this investigation were obtained from Karimi & Naderi (2008), who conducted an experiment at Karaj Research Center, Karaj, I. R. of Iran in 2003. Table 1 shows the soil physical and chemical properties (Karimi & Naderi, 2008). The experiment was a split-plot design (1 plot = 3×6 m^2) with randomized complete blocks arrangement with three replications. Irrigation treatments were the main plots and N fertilizer rates were subplots. Four levels of irrigation water (40%, 80%, 120% and 160% of evaporation from the surface of class A evaporation pan) and four of N fertilizer (urea) (0, 90, 180 and 270 kg N/ha) were applied. Irrigation frequency was the same for all treatments (every 7 days). Sum of the applied water and seasonal rainfall for different irrigation treatments were 9500, 11850, 14500 and 16450 m^3/ha, respectively. Seasonal rainfall was 207 m^3/ha. MSC2 cultivar of sugar beet was planted. Seeds were planted on rows with spacing between rows of 0.6 m and distance between seeds on rows of 0.2 m. After harvest, the sugar concentration was determined by standard procedures by the sugar refining factory. The soil residual mineralized nitrogen (NO_3 and NH_4) of the root zone was considered as 255.15 kg/ha for two soil layers (5% of the soil total N (Table 1), as reported by Dhanke & Vass, 1973).

Table 1. Soil physical and chemical properties of the experimental site (from Karimi & Naderi, 2008).

Properties	Soil depth (cm)	
	0-30	30-60
Field capacity (g/g)	0.19	0.18
Permanent wilting point (g/g)	0.1	0.09
Bulk density (g/cm)	1.38	1.47
Soil saturation extract salinity (dS/m)	0.76	0.52
pH	7.4	7.7
Organic matter (%)	0.52	0.34
Total nitrogen (%)	0.07	0.05
Soil texture	Clay loam	Clay loam

Rainfall probability of occurrence

Using the above-mentioned analysis to plan the deficit irrigation is contingent on the prediction of seasonal rainfall amount before the start of the growing season or on the amount of optimum applied water that should be determined by using occurrence probability analysis for a given rainfall (Sepaskhah *et al.*, 2008).

Rainfall amounts during the growing season for 21 years in the study area are available (http://www.irimo.ir). A frequency analysis was applied for the amounts of optimum applied water at different N levels obtained in different years. The occurrence probability was estimated by Weibull equation (Chow *et al.*, 1988) as follows:

$$p = \frac{m}{n+1} \quad (12)$$

where p is the probability of occurrence in fraction, m is the rank of observation, and n is the number of observations.

Crop yield, cost and crop price function

Based on the sugar beet root yield, sugar content, sum of irrigation water and rainfall and sum of N application rate and residual N, reported by Karimi & Naderi (2008), crop yield and sugar concentration functions were determined by multiple regression analysis as follows:

$$y = 32805.86 - 2.66647(W+R) - 97.746(N+Nr) + 0.023134(W+R)(N+Nr) - 1.12*10^{-9}(W+R)^2(N+Nr)^2 \quad (13)$$

n=16, SE=866.08, p<0.001, R^2=0.987

$$SC = 17.828 - 1.3926*10^{-8}(W+R)^2 - 1.8502*10^{-6}(N+Nr)^2 \quad (14)$$

n=16, SE=0.664, p<0.001, R^2=0.72

In Eqns. (13) and (14), units of y, N, and Nr are kg/ha; W and R, m^3/ha; and SC, %.

As mentioned above, crop price (P_c) depends on root sugar concentration. Therefore, crop price (P_c) is a function of W+R and N+Nr and its value was determined as follows (Tavakoli & Fardad, 1999):

$$P_c = (SC-3)*P_{C16}/13 \quad (15)$$

where P_{c16} is the base price of sugar beet (Rls/kg) with SC of 16% and its value was 290 Rls/kg at the time of carrying out the experiment. Values of 3 and 13 are a mean waste (%) and mean sugar concentration (%) in previous years, respectively, as suggested by the Iranian Agricultural Ministry (Tavakoli & Fardad, 1999). The cost function is the sum of fixed cost including land preparation, seeding, herbicides and pesticides, harvest, and land rent and variable costs consisted of water cost (P_w=120 Rls/m^3) and N fertilizer cost (P_n =913 Rls/kg) as follows:

$$C = 3024631 + 120W + 913N \quad (16)$$

Results

Maximum yield

Solving Eqns (11a) and (11b) based on the coefficients of Eqn (13), the amount of applied water (W_m) and nitrogen (N_m) that resulted in maximum yield was obtained as follows:

$$W_m + \mathrm{R} = 16871.5 \Rightarrow \text{ for } R = 207 \text{ m}^3/\text{ha}, \quad W_m = 16664.5 \text{ m}^3/\text{ha} \quad (17a)$$

$$N_m + \mathrm{Nr} = 460.2 \Rightarrow \text{ for } Nr = 255.15 \text{ kg/ha}, \quad N_m = 205.05 \text{ kg/ha} \quad (17b)$$

Land limiting conditions

Under land limiting conditions, saved water compared to W_m cannot be used to irrigate extra land area. This amount of water maximized the benefit for each unit of land. By solving Eqns (7a) and (7b) based on yield [Eqn (13)], crop price [Eqn (15)] and cost [Eqn (16)] functions for R=207 m^3/ha, Nr=255.15 kg/ha, P_{c16}=290 Rls/kg, P_n=913 Rls/kg and P_w=120 Rls/m^3, the calculated optimum water (W_l) and N (N_l) under land limiting conditions were 12174.2 m^3/ha and 262.6 kg/ha, respectively. For more assessments, W_l and N_l were calculated at different rainfalls, water costs, N cost, base crop prices and soil residual N. Results are shown in Figs. 1 and 2 which indicate that as rainfall increased, W_l decreased, whereas N_l did not vary (Fig. 1a). Therefore, rainfall had not any effect on the optimal amount of N_l. For constant N cost (P_n =913 Rls/kg) and base crop price (P_{c16}=290 Rls/kg), by increasing water cost (P_w), optimum amounts of water and N decreased and increased, respectively (Fig. 1a). For constant water cost (P_w=120 Rls/m^3) and base crop price (P_{c16}=290 Rls/kg), by increasing N cost (P_n), the optimum amount of N decreased; however, the optimum amount of water increased (Fig. 1b). For a given rainfall, the optimum level of water increased by an increase in base crop price (Fig. 1c). In other words, maximum net income was found at a higher depth of water by an increase in base crop price. For a given rainfall, N_l variation was very low by a change in base crop price (Fig. 1c) and was higher for a higher water and N cost (Figs. 1a and 1b). The value of W_l was not affected by soil residual N content, and the value of N_l decreased by an increase in Nr (Fig. 2).

Net income was calculated based on different irrigation water and N applications at different water and N costs and base crop price for constant rainfall and soil residual N. Relationships between net income and irrigation water at different water costs and base crop prices for R=207 m^3/ha and N+Nr=517 kg/ha are shown in Fig. 3. As mentioned above, the maximum net income per unit area (the maximum points) occurred at optimum irrigation water for different water costs (Fig. 3a). An increase in water cost decreased net income. Also, maximum net income was obtained at lower irrigation water depth by the increase in water cost compared with P_w=0 (arrow in Fig. 3a). Maximum net income increased by increase in the base crop price. The maximum net income was obtained at higher irrigation water by an increase in base crop price (arrow in Fig. 3b). The net income under land limiting conditions at different N costs for P_{c16}=290 Rls/kg and W+R= 12381 m^3/ha are shown in Fig. 4. The maximum net income per unit area (maximum point) occurred at optimum N application. The maximum net income decreased and occurred at a lower N level by an increase in N cost (arrow in Fig. 4).

Water limiting conditions

For an optimum amount of water under water limiting conditions, the saved water with respect to W_m can be used to increase the A (more planting area). Therefore, net income gained from total A is increased. By solving Eqns (10a) and (10b) based on yield [Eqn (13)], crop price [Eqn (15)] and cost [Eqn (16)] functions, for R=207 m^3/ha, Nr=255.15 kg/ha, P_{c16}=290 Rls/kg, P_n=913 Rls/kg and P_w=120 Rls/m^3, the calculated optimum water (W_w) and N (N_w) under water limiting conditions were 8410.6 m^3/ha and 300.7 kg/ha, respectively.

For more assessments, the values of W_w and N_w were calculated at different rainfall amounts, water costs, N costs, base crop prices and soil residual N contents. The results are shown in Figs. 5 and 6. By an increase in rainfall (Fig. 5), residual N (Fig. 6), water cost (Fig. 5a) and base crop price (Fig. 5c), the value of applied water decreased. The value of W_w increased by an increase in the N cost (Fig. 5b). For N_w, with exception of base crop price (Fig. 5c), the

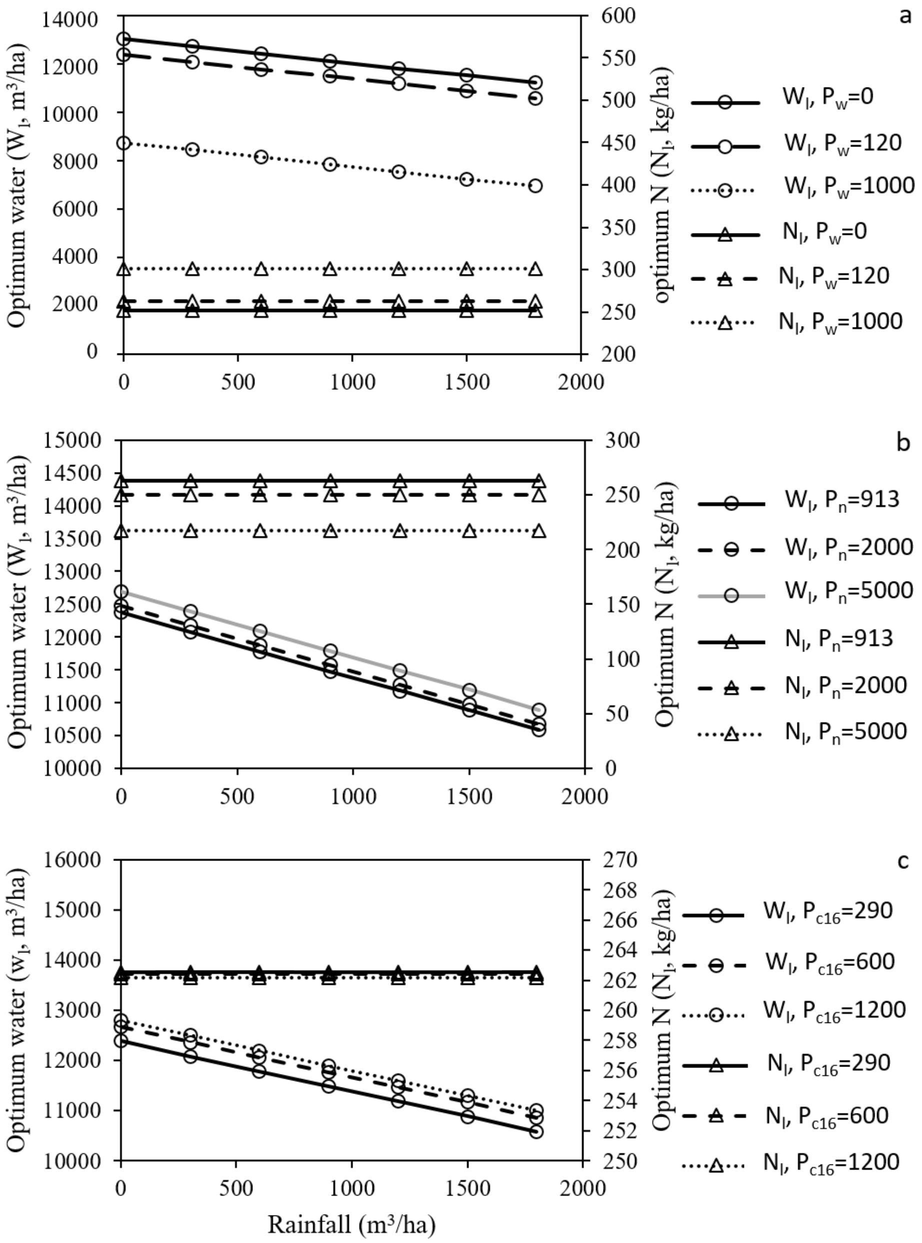

Figure 1. Relationship between the optimum water (W_l) and nitrogen (N_l) under land limiting conditions and rainfall at different water costs [a, P_w (Rls/m^3)], N costs [b, P_n (Rls/kg)] and base crop prices [c, P_{c16} (Rls/kg)].

value of N_w decreased by increase in rainfall (Fig. 5), soil residual N content (Fig. 6), water cost (Fig. 5a), and N cost (Fig. 5b). For a constant water cost and base crop price, by increasing N cost, the optimum N decreased, and optimum water increased (Fig. 5b). Net income decreased by an increase in N cost. Therefore, the maximum net income per unit area occurred at a higher water level in which yield was higher. For a given rainfall, the optimum level of applied water decreased by an increase in base crop price. In other words, maximum net income occurred at lower applied water by an increase in base crop price (Fig. 5c).

The net income under conditions of water limiting for seasonal rainfall of 20.7 mm and N+Nr=517 kg/ha at different water costs and base crop prices are shown in Fig. 7.

Optimum water, N, grain yield, net income and land increase for Nr=255 kg/ha, P_w=120 Rls/m^3, P_n= 913 Rls/kg and P_{c16}=290 Rls/kg at different rainfall depths were calculated and presented in Table 2. Rainfall occurrence resulted in increase in the amount of total net income due to a decrease in water cost. In Fig. 7, the maximum points occurred at irrigation depth that is equal to W_w. The maximum point was shifted to the left

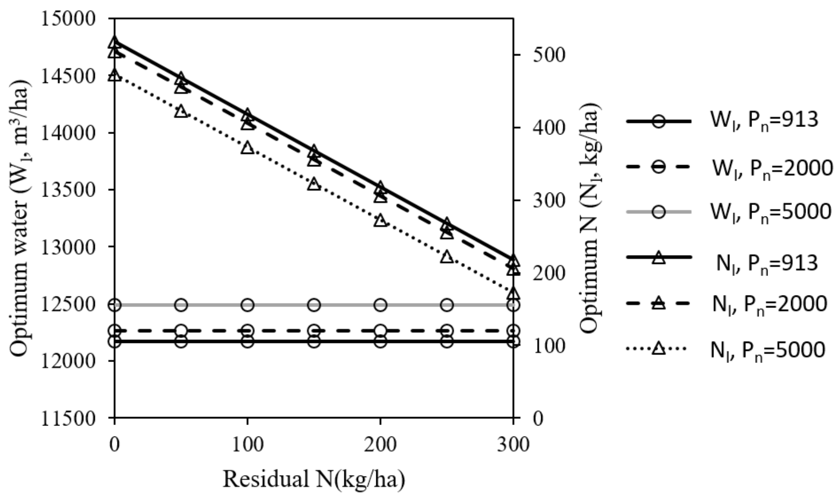

Figure 2. Relationship between the optimum water (W_l) and nitrogen (N_l) under land limiting conditions and residual N at different N costs (P_n, Rls/kg) for rainfall=207 m^3/ha, base crop prices (P_{c16}) =290 Rls/kg and water price (P_w)=120 Rls/m^3.

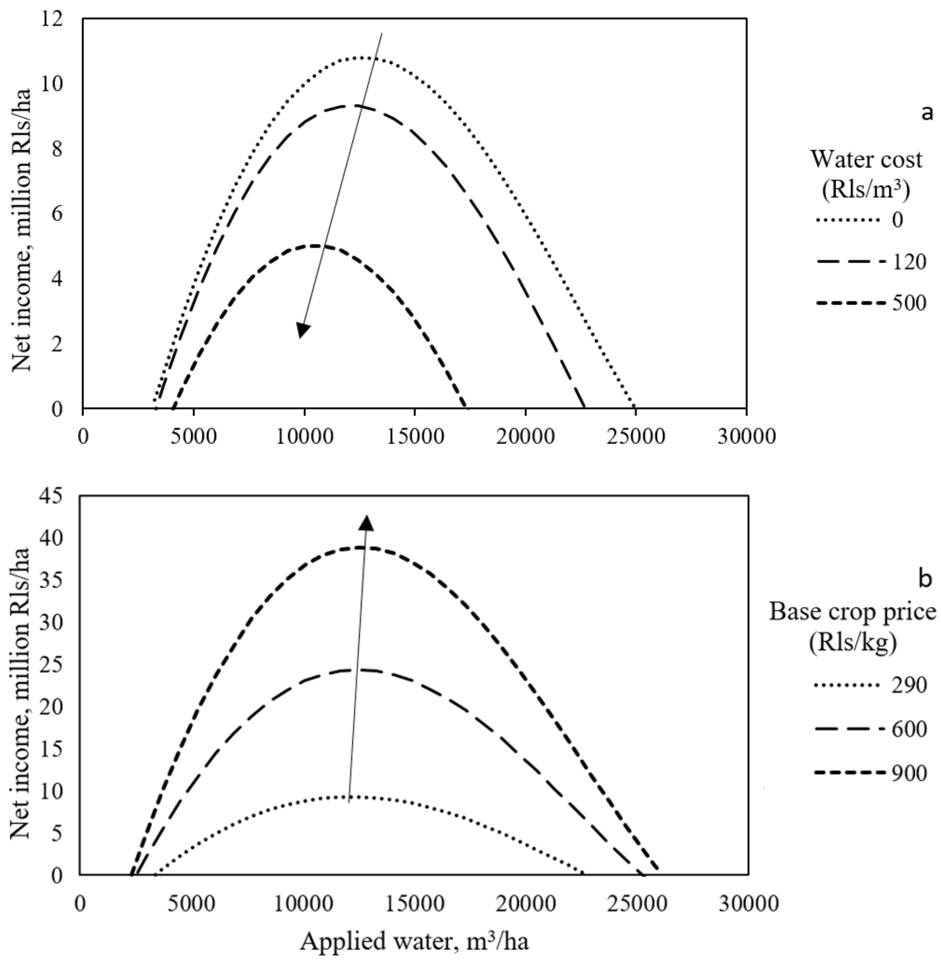

Figure 3. Relationships between the net income and irrigation water under land limiting conditions at different water cost (a) and base crop price (b) for sum of the N and soil residual mineral N (N+Nr)=517 kg/ha and rainfall=20.7 mm. Arrows cross through maximum net income.

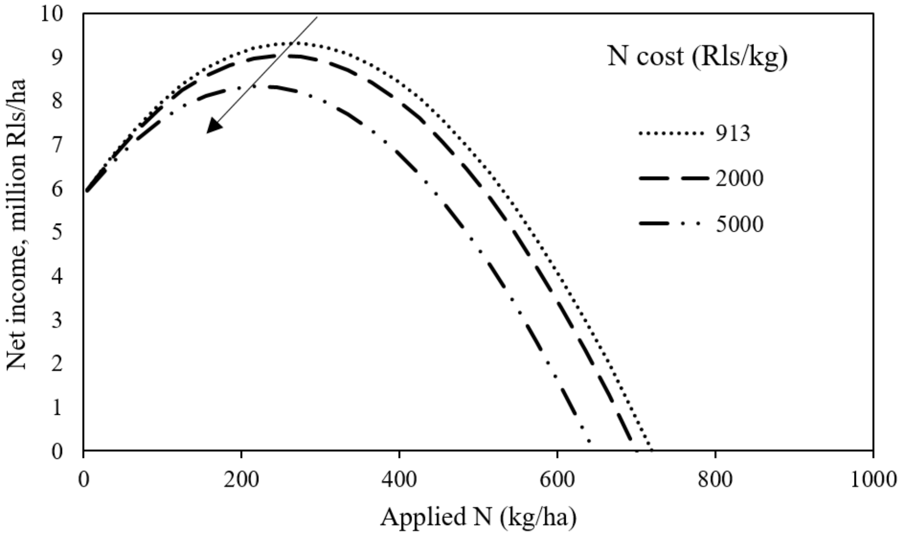

Figure 4. Relationships between the net income and applied N under land limiting conditions at different N cost for base crop price (P_{c16})=290 Rls/kg and sum of irrigation water and rainfall (W+R)=12381 m^3/ha. The arrow crosses the peak points.

and optimum irrigation water decreased by an increase in the water cost and base crop price (arrows in Fig. 7). The net income under water limiting conditions at different N costs for P_{c16}=290 Rls/kg and W+R=12381 m^3/ha are shown in Fig. 8. Maximum net income in the total irrigated area (maximum point) occurred at optimum N application. The maximum net income decreased and occurred at a lower N level by increase in N cost due to increase in production costs (arrow in Fig. 8).

Constant crop price conditions

When the crop price is fix (it does not change with crop quality), its value will be determined based on supply and demand system or government pricing. For fixed crop price, term $pc(W+R, N+Nr)$ in Eq. (2) changed to a constant value (P_c). Anyway, the fixed crop price may be either higher than the variable crop price or lower than it. Therefore, the results of these two situations are similar to those obtained by changing the base crop price in Eqn. (15). For example, as presented in Table 2, optimum water, optimum N, grain yield, net income and land increase for Nr=255 kg/ha, P_w=120 Rls/m and P_n=913 Rls/kg in case of the fixed crop price and equal to the base crop price (P_c= P_{c16}=290 Rls/kg) at different rainfall depths were calculated. In other words, crop price did not change with crop quality and it was assumed that the value of SC in Eqn. (15) was 16 and P_c= P_{c16}=290 Rls/kg. In these conditions, crop price was higher than the varied crop price due to root sugar concentration lower than 16% (mean sugar content is 13% in Iran; Tavakoli & Fardad, 1999). As mentioned above, increasing of P_{c16}, to higher crop price under constant crop price conditions resulted in higher W_l, W_w, N_w and total net income and lower N_l compared with the crop price under varied crop prices (Table 2).

Deficit irrigation planning

For planning deficit irrigation, the amounts of optimum applied water at different N levels were obtained for 21 years of seasonal rainfall values in the study area. A frequency analysis was applied for the amounts of optimum applied water at different N levels obtained in different years. The occurrence probability for the values of optimum applied water was estimated by the Weibull equation [Eqn (12)]. The suitable distribution function was Gumbel (Fig. 9) based on the Kolmogorov-Smirnov index of goodness of fit. Results of distribution function showed that under water limiting conditions, the probability of occurrence for W_w as 50% and 80% that is corresponding to 3001.9 and 3400.3 m^3/ha of water for N_w=0.0 kg/ha, 7135.6 and 7430.3 m^3/ha of water for N_w=100.0 kg/ha and 7996.0 and 8278.9 m^3/ha of water for N_w=200.0 kg/ha, respectively, could be used in irrigation water resources planning, which is in accordance with the probability of occurrence of 50% and 80% for seasonal rainfall.

Discussion

In the current study, required equations for determining the optimum applied irrigation water depth

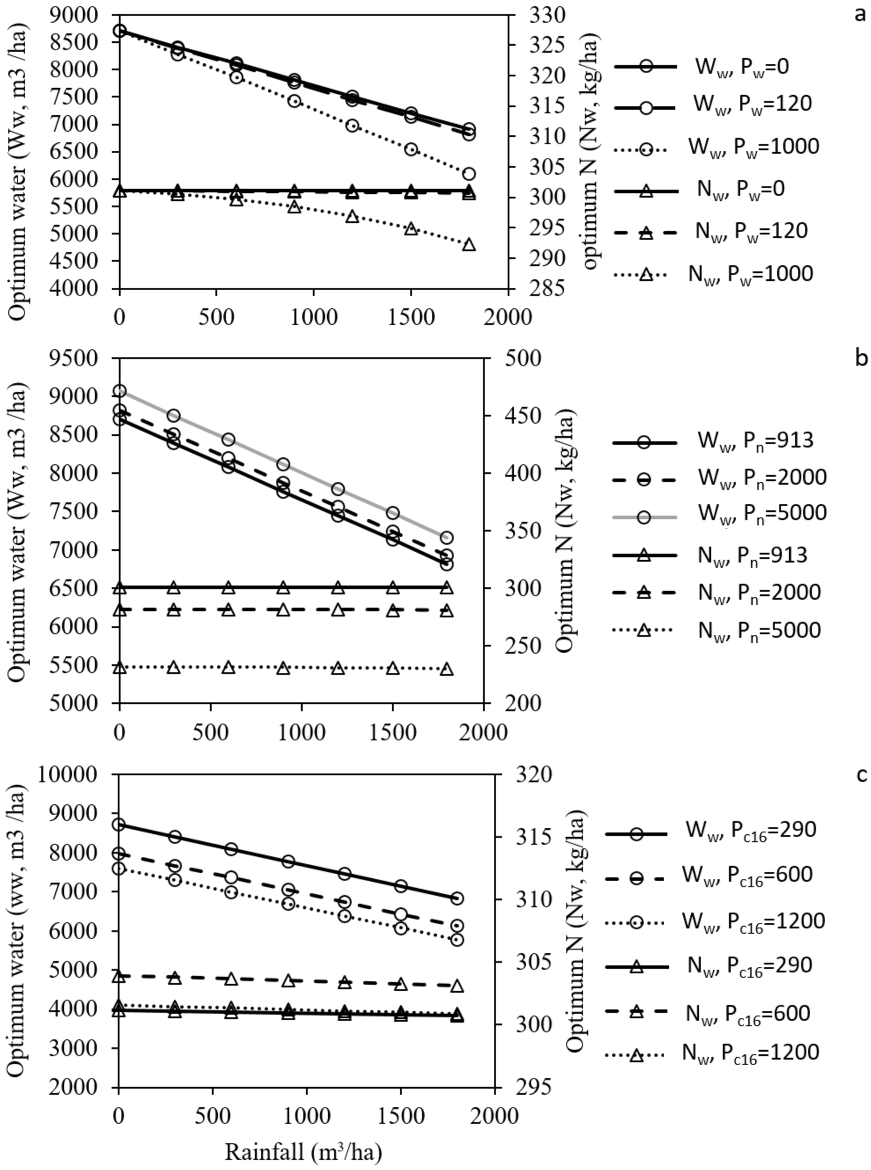

Figure 5. Relationship between the optimum water (W_w) and nitrogen (N_w) under water limiting conditions and rainfall at different: (a) water costs [P_w (Rls/m^3)], (b) N costs [P_n (Rls/kg)] and (c) base crop prices [P_{c16} (Rls/kg)].

and N fertilizer under full irrigation, limited land and water conditions for sugar beet were derived when the crop price is a function of the sum of irrigation water and seasonal rainfall and N fertilizer. Results showed that applied water and N to maximize yield decreased by an increase in rainfall and soil residual mineral N, respectively. The higher rainfall supplied, the higher portion of crop water requirements. These results are similar to those obtained by Sepaskhah *et al.* (2008) for saffron and Sepaskhah *et al.* (2006) for winter wheat.

Under land limiting conditions by an increase in rainfall and water price, optimum water decreased due to partly supplying of the crop water requirement by rainfall. Similar results were reported by Oweis & Hachum (2006) and Amiri *et al.* (2016) for rice and wheat, respectively. A decrease in N cost and base crop price resulted in decrease in W_l as shown by Zand-Parsa & Sepaskhah (2001) under land limiting conditions. The value of W_l was not affected by Nr. Similarly to the results obtained by Sepaskhah *et al.* (2006), the applied N under land limiting conditions decreased by an increase in Nr. The value of N_l increased by an increase in water price and a decrease in base crop price and N cost. Net income decreased by an increase in water and N costs due to the increase in production costs. Therefore, the maximum net income per unit area occurred at higher water and N levels in which yield was higher.

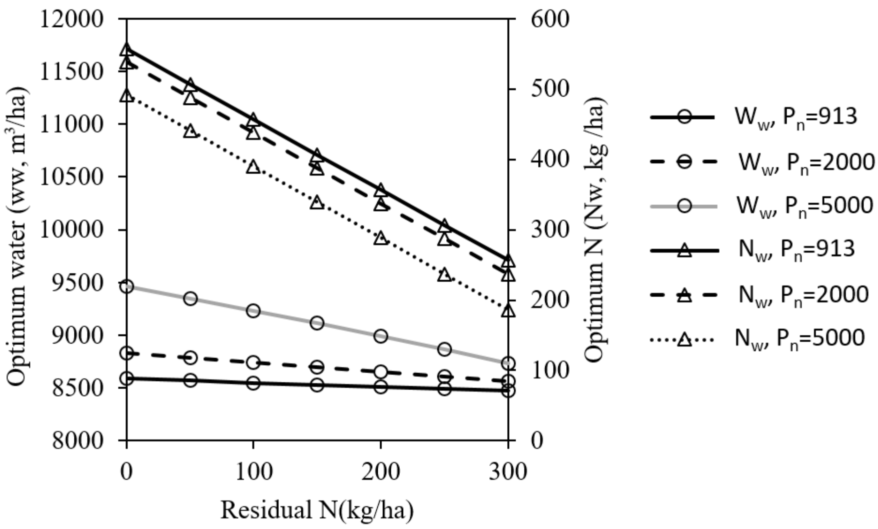

Figure 6. Relationship between the optimum water (W_w) and nitrogen (N_w) under water limiting conditions and residual N at different N costs (P_n, Rls/kg) for rainfall=207 m^3/ha, base crop prices (P_{c16}) =290 Rls/kg and water price (P_w)=120 Rls/m^3.

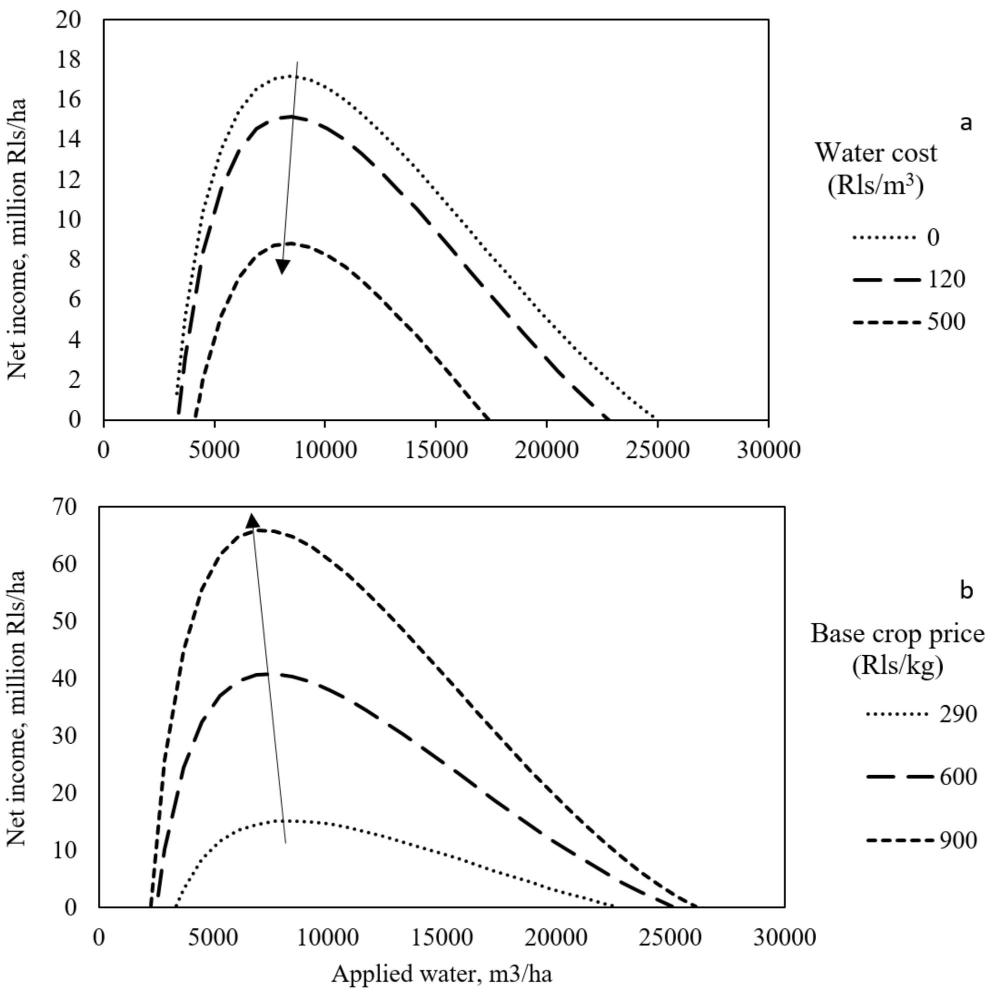

Figure 7. Relationships between the net income and irrigation water under water limiting conditions at different: (a) water costs and (b) base crop prices for sum of the N and soil residual mineral N (N+Nr)=517 kg/ha and rainfall=20.7 mm. Arrows cross through maximum net income.

Table 2. Optimum water, nitrogen, grain yield, net income and land increasing for soil residual mineral N =255 kg/ha, water price =120 Rls/m³ and N fertilizer cost =913 Rls/kg at different rainfall depths.

Seasonal rainfall (cm)	Optimum water (m³)	Optimum N (kg/ha)	Yield (t/ha)	Net income per unit area (10⁶ Rls/ha)	Net income per unit water (Rls/m³)	Land increase (%)	Total net income (10⁶ Rls)
Variable crop price (base crop price =290 Rls/kg)							
Maximum yield							
0	16871.5	205	55.1	7.65	453.2	—	7.65
6	16271.5	205	55.1	7.72	474.3	—	7.72
12	15671.5	205	55.1	7.79	497.1	—	7.79
Land limiting							
0	12381.2	262.5	51.6	9.29	750.3	—	9.29
6	11781.2	262.5	51.6	9.36	794.7	—	9.36
12	11181.2	262.5	51.6	9.43	843.7	—	9.43
Water limiting							
0	8708.1	301.1	41.1	7.75	890.0	93.7	15.01
6	7828.8	299.5	39.9	7.56	966.1	108	15.71
12	6882.1	295.5	38.3	7.28	1058.4	128	16.59
Fixed crop price (crop price =290 Rls/kg)							
Maximum yield							
0	16871.5	205	55.1	10.76	637.5	----	10.76
6	16271.5	205	55.1	10.83	665.4	----	10.83
12	15671.5	205	55.1	10.9	695.5	----	10.9
Land limiting							
0	15643.6	222.4	54.9	10.82	691	----	10.82
6	15043.6	222.4	54.9	10.9	724	----	10.9
12	14443.6	222.4	54.9	10.97	759	----	10.97
Water limiting							
0	9676.1	332.9	45	8.563	885	74.4	14.93
6	8768.4	335.9	43.9	8.352	952.5	92.4	16.07
12	7810.4	338.1	42.5	8.069	1033.1	116	17.42

1 US$ ~ 8280 Iranian Rials (at the time of the experiment).

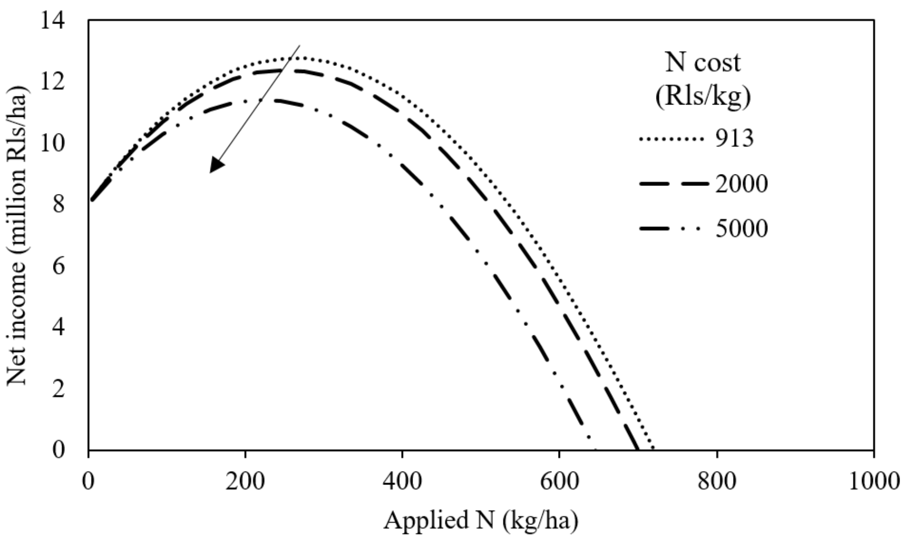

Figure 8. Relationships between the net income and applied N under water limiting conditions at different N costs for base crop price (P_{c16})=290 Rls/kg and sum of irrigation water and rainfall (W+R)=12381 m³/ha. The arrow crosses the peak points.

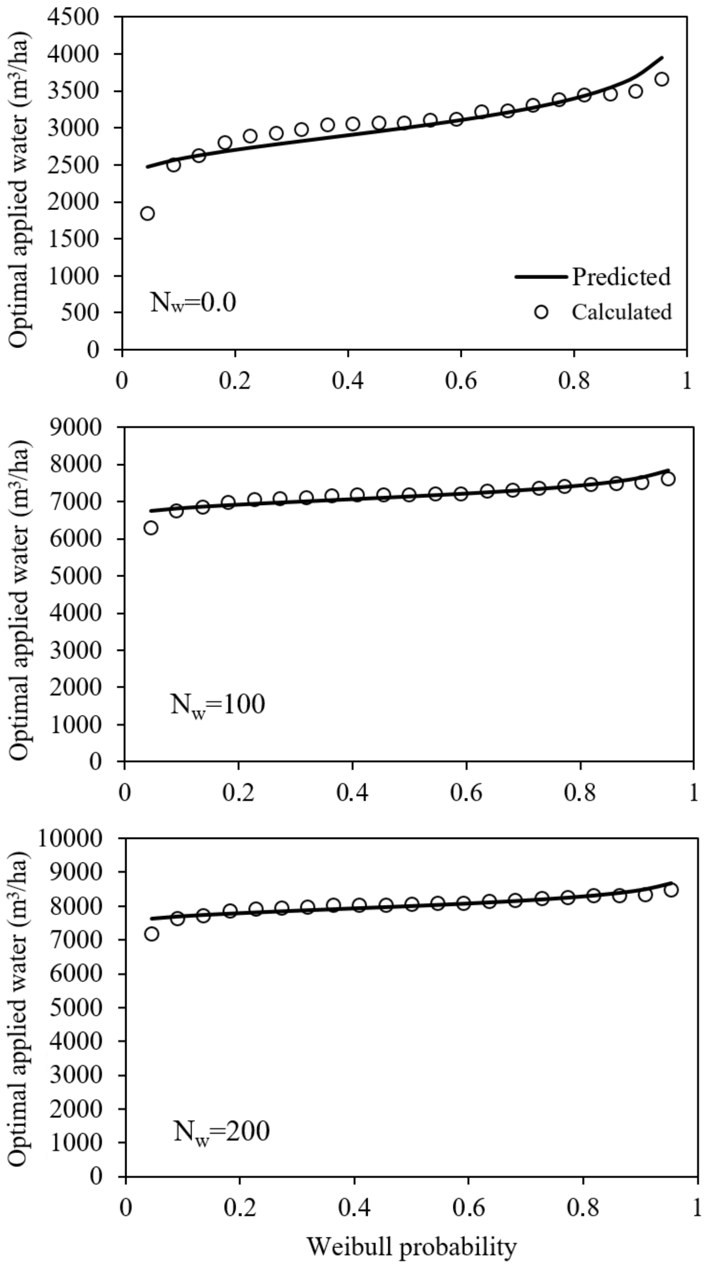

Figure 9. Relationships between the optimum applied water and Weibull probability under limiting water conditions at the different N level (N_w, kg/ha) for soil residual mineral N (Nr)=255 kg/ha, water price (P_w)=120 Rls/m^3, N fertilizer cost (P_n)=913 Rls/kg and base crop price (P_{c16})=290 Rls/kg.

Under limited water conditions, by an increase in rainfall, soil residual N, water cost and base crop price, the value of optimum applied water decreased (Zhang & Oweis, 1999; Zand-Parsa & Sepaskhah, 2001; Amiri *et al.*, 2016; Khozaie & Sepaskhah, 2018). The value of W_w increased by an increase in the N cost. With the exception of base crop price, the value of optimum N under water limiting conditions decreased by an increase in rainfall, soil residual N, water cost, and N cost. As discussed by Sepaskhah *et al.* (2006), in the conditions of limited water, the cultivated land under irrigation is determined as W_m/W_w. In this condition, as obtained in the current study, the net income is maximized at less applied water in comparison with that obtained at limited land conditions. Under limited water conditions, the net benefit per unit water is maximized, while under limited land conditions, the net benefit per unit land is maximized. According to English & Raja (1996), Sepaskhah & Akbari (2005) and Shabani *et al.* (2018), results showed that the net income per unit water was higher under limited water conditions compared to the maximum yield and limited land conditions. However, yield and net income per unit area were low. Therefore, the total obtained net income from all irrigated areas increased due to the increase in cultivated area (Pereira *et al.*, 2002). Increase in water cost and base crop price decreased and increased the net income due to an increase in production cost and income, respectively (Rey *et al.*, 2016).

Optimum N application under limited land and limited water conditions was higher than its value under full irrigation due to the fact that N fertilizer application was not limited and maximum net income per unit area and water occurred at higher N level in which yield was higher (Zand-Parsa & Sepaskhah, 2001; Sepaskhah *et al.*, 2006). In these conditions, N leaching decreased as a result of a decrease in applied water and drainage water. Therefore, using these optimum water depth and N create lower environmental pollution.

For no rainfall in the growing season, deficit irrigation as 27% and 48% compared to full irrigation resulted in 6.4% and 25.4% decrease in yield and 21.4% and 96.2% increase in total net income under land and water limiting conditions, respectively. Under water limiting conditions, for given base crop price, water cost, N cost and soil residual N, cultivated land area increased by 93.7, 108 and 128% for 0, 60 and 120 mm rainfall, respectively.

For fixed crop price, derived equations in this study is similar to the derived equations by Zand-Parsa & Sepaskhah (2001) and Sepaskhah *et al.* (2006) and increasing of base crop price resulted in increase in W_l, W_w, N_w and total net income and decrease in N_l.

The economic-mathematical analysis presented here can be used for other regions and crops, e.g., sugarcane and other crops for which the crop price is dependent on yield quality. Yield and crop price functions used in the current study were empirical and should be determined for specific climates and cultivars.

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