

# Improving the efficiency of spatially selective operations for agricultural robotics in cropping field

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## Abstract

Cropping fields often have well-defined poor-performing patches due to spatial and temporal variability. In an attempt to increase crop performance on poor patches, spatially selective field operations may be performed by agricultural robotics to apply additional inputs with targeted requirements. This paper addresses the route planning problem for an agricultural robot that has to treat some poor-patches in a field with row crops, with respect to the minimization of the total non-working distance travelled during headland turnings and in-field travel distance. The traversal of patches in the field is expressed as the traversal of a mixed weighted graph, and then the problem of finding an optimal patch sequence is formulated as an asymmetric traveling salesman problem and solved by the parthenogenetic algorithm. The proposed method is applied on a cropping field located in Northwestern China. Research results show that by using optimum patch sequences, the total non-working distance travelled during headland turnings and in-field travel distance can be reduced. But the savings on the non-working distance inside the field interior depend on the size and location of patches in the field, and the introduction of agricultural robotics is beneficial to increase field efficiency.

**Additional key words:** ASTP; autonomous machines; field operations; fieldwork pattern; patch spraying; precision agriculture; spatially selective application.

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## Introduction

Spatially selective field operations is a term introduced by Stafford & Ambler (1991) to describe the field operations approach of targeting inputs to arable crops according to locally determined requirements (Stafford, 1996), *i.e.*, applying agronomic inputs by modern machines in the right place, at the right time and in the right quantity to improve the economic efficiency, and to diminish the environmental impact of crop production (Earl *et al.*, 2000). Recently, the increased availability and reduced cost of information technology and electronic control systems have made this concept more practical (Paice *et al.*, 1996). Actually, due to the spatio-temporal variability in factors such as soil type, nutrient availability and crop pest, cropping fields (Oliver & Robertson *et al.*, 2010) often have poor-performing patches dispersedly covering less than 40%

of the area (Earl *et al.*, 2000). In an attempt to increase crop performance (*i.e.*, crop yield) on poor patches, farm managers or contractors may apply additional inputs by modern machines (Oliver *et al.*, 2010). But its implementation is subject to the crop rows and the location of sparsely located patches, thus the pattern of field operation affects very much the time lost in the field due to the non-working travel stems from the turning at headlands (Taylor *et al.*, 2002; Palmer *et al.*, 2003; Bochtis & Vougioukas, 2008), as well as the non-productive transition inside the field interior. Therefore, an optimal fieldwork pattern is essential to improve the efficiency of such operations.

Recently, efficiency studies have been directed towards optimizing the headland maneuvers and travel sequences of autonomous agricultural machines. For automation of maneuvers in headlands, some approaches in terms of maneuvers generation and vehicle control have been

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Abbreviations used: ATSP (asymmetric traveling salesman problem); AU (application unit); GIS (geographical information system); GPS (global positioning system); RS (remote sensing); RU (refilling unit).

proposed and mainly based on loop turns and reverse turns (Oksanen & Visala, 2004; Cariou *et al.*, 2009; 2010). In terms of computing traversal sequences various coverage path planning algorithms were proposed, *e.g.*, genetic algorithm (Ryerson & Zhang, 2007), ant colony algorithm (Bakhtiari *et al.*, 2011) and other combinatorial optimization algorithm. In particular, numerous researchers have formulated the path planning by graph abstraction, such as the Chinese Postman Problem (Sørensen *et al.*, 2004), the traveling salesman problem (Bochtis & Vougioukas, 2008) and the modified minimum cost network flow problem (Ali *et al.*, 2009). Although the optimization of fieldwork pattern for field efficiency has been investigated extensively in the literature, the developed approaches (*e.g.*, typically proposed by Bochtis & Vougioukas in 2008) are applicable to completely covering the whole field and for minimizing the non-working travelled distance during headland turnings only, which cannot be used without modification for spatially selective field operations because of the non-working travel inside the field interior may constitute the determining factor for field efficiency during such operations. Furthermore, in the context of precision farming, research in terms of autonomous vehicle navigation has focused mainly on the accurate tracking of predetermined paths. The problem of automatically computing the path of such operations has not received as much attention (Vougioukas *et al.*, 2006).

In order to address the route planning problem for an agricultural robot that has to treat some poor-patches in a given field with row crops, the primary goal of this paper is to present an algorithmic approach which minimizes the total non-working distance travelled during headland turnings and in-field travel distance, whilst to analyze the influence of the size and location of patches in the field with respect to the savings on the non-working distance.

## Material and methods

In this section, the traversal of patches in the field's interior is first expressed as the traversal of a mixed weighted graph. Then, the problem of finding an optimal traversal sequence is formulated as an asymmetric traveling salesman problem (ATSP) and solved by the partheno-genetic algorithm. Finally, an illustrative example is designed to demonstrate the effectiveness of the proposed approach.

## The mathematical model

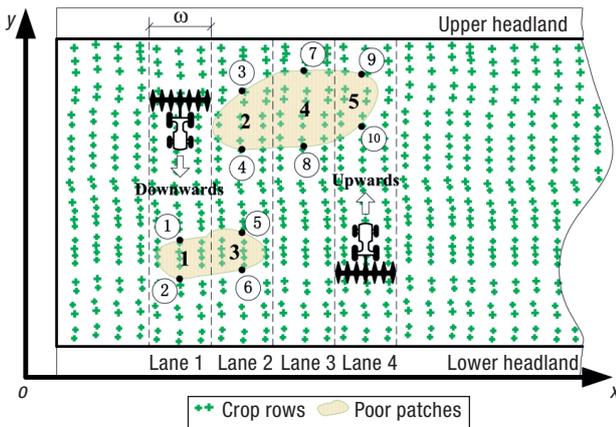
### *Preliminaries and assumptions*

The scenario to be studied involves a cropping field covered by a set of parallel crop rows which starts at one headland of the field and terminates at the opposite headland. Some poor patches (or zone) are sparsely located within the field because of the spatially heterogeneous soils and crop performance. In an attempt to increase crop performance (*i.e.*, crop yield) on poor patches, the spatially selective field operations may be implemented to apply additional fertilizer or ameliorants according to targeted requirements. For practical purpose, in this work some assumptions and limitations are first set: (i) the agricultural field operations are executed by the machinery system that uses an in-field operating machine application unit (AU) cooperating with an out-of-the field transport unit refilling unit (RU); (ii) the autonomous agricultural machine used within the field is considered as an agricultural robot able to move forward or backward along a crop row; (iii) the refilling of the AU tank can be performed just in time by the refilling unit (RU) located outside the field when the autonomous agricultural machine reached a headland; (iv) there are no obstacles in a given field region.

### *Definitions of vectors and functions*

For sake of mathematical description of the spatially selective field operations, a 2-dimensional coordinate system  $(x,y)$  as Bochtis *et al.* (2009) is assigned to the field, where the  $y$ -axis is parallel to the crop rows (which are assumed to be straight line or approximate one). As shown in Fig. 1, consider a cropping field including two headlands: "upper" headland and "lower" headland. The "upper" headland is located in the positive direction of the  $y$ -axis, while the "lower" headland is located in the negative direction. Therefore, when the application unit (AU) is moving parallel in the positive direction of the  $y$ -axis, it is considered to be moving "upwards", while moving in the opposite direction it is considered to be moving "downwards" (Bochtis *et al.*, 2009).

As soon as the poor patches are defined via precision agriculture technologies (*e.g.*, RS, GPS, GIS, etc.), the crop rows that have patches can be converted into a series of parallel lanes according to the operating width of agricultural robotics. Let  $K = \{1,2,3,\dots,|K|\}$



**Figure 1.** A coordinate system concerning the application unit (AU) operating motion and the patch numbering.

be the ordered set of field lane indices where the value of the lane indices increases towards the positive direction of the  $x$ -axis (Fig. 1). The intersection of a lane with the upper headland will be described as the “upper” ending of the lane, while the other ending will be described as the “lower” ending (Bochtis *et al.*, 2009). The total number of lanes with patches in the whole field is given by the cardinality of the set  $K$  (Bochtis & Vougioukas, 2008):

$$\|K\| = \left\lceil \frac{l}{\omega} \right\rceil \quad [1]$$

where  $l$  is the total number of crop rows with poor patches,  $\omega$  is the operating width of agricultural robotics, and the symbol  $\lceil \cdot \rceil$  denotes the ceiling function.

In order to present the location of each patch, some other sets are defined as following:  $M = \{1, 2, 3, \dots\}$  is the set of the patch indices of all lanes;  $T_1 = \bigcup_{m \in M} (2m - 1)$  is the set of the upper ending indices of all patches;  $T_2 = \bigcup_{m \in M} (2m)$  is the set of the lower ending indices of all patches;  $T = T_1 \cup T_2$  is the set of the ending indices of all patches.

Let  $V_k^{index} (x_k^{index}, y_k^{index})$ ,  $k \in K$  be the vector of the coordinates of the upper and lower endings of the patch ( $index \equiv t$ ,  $t \in T$ ), or that of the upper ending ( $index \equiv u$ ) and lower ending ( $index \equiv d$ ) of the lane  $k$ . The bijective function  $p(\cdot) : T \rightarrow T$  was introduced by Bochtis & Vougioukas (2008). To every patch,  $t \in T$ , the function  $p(t)$  returns the order in which the machine covers the patch  $t$  in the field. Therefore, the travel sequence for covering all patches can be given by the permutation,  $\mu = \langle p^{-1}(1), p^{-1}(2), \dots, p^{-1}(\|T\|) \rangle$ . Let  $f_1(\cdot) : T \rightarrow K$ ,  $f_2(\cdot) : T \rightarrow K$  be the surjective function, such that for every element of one set there is a unique element of

another set. Finally, the operator  $\Delta : \mathbb{R}^2 \times \mathbb{R}^2 \rightarrow \mathbb{R}$  is defined as Bochtis *et al.* (2009), which returns the Euclidian distance between two points belonging to the same lane for given the vectors of their coordinates, e.g., the length of the lane  $k$  is then equal to  $\Delta(V_k^u, V_k^d)$ .

### Calculation of non-working travel distances

The non-working distances during a tour mainly include headland turnings and in-field transition from one patch to another. Some common maneuvers for any agricultural machine operating in a headland pattern include the loop, or forward-turn ( $\Omega$ -turn), the double round corner ( $\Pi$ -turn) and its variant the reverse or switch-back-turn ( $T$ -turn). Due to a long travelled distance and the larger lateral forces that is required to execute  $\Omega$ -turns, the absence of this kind of turns is beneficial to decrease the travel distances (Bochtis & Vougioukas, 2008). Therefore only  $\Pi$ -turn and its variant are considered in this work. Additionally, consider the case that the AU may be moving backward along a crop row for exiting a lane and then be moving forward in the headland area for entering a new lane; we termed this kind of turning as  $R$ -turn for description convenient. Then the turning distance of maneuvers mentioned above for the transition of the AU from the exit-point of lane  $h$  to the entry-point of lane  $k$  can be computed approximatively by (Bochtis *et al.*, 2009):

$$\begin{aligned} \Pi(|k-h|) &= (\pi - 2)r_{min} + |k-h| \cdot \omega \\ T(|k-h|) &= (\pi + 2)r_{min} + |k-h| \cdot \omega \\ R(|k-h|) &= \pi r_{min}/2 + |k-h| \cdot \omega \end{aligned} \quad [2]$$

where  $r_{min}$  is the minimum turning radius of the machine,  $|k-h|$  is the length of the headland path that connects the two lanes  $h$  and  $k$ .

In some situations, the length of lanes is different, that is, headlands do not constitute rectilinear segments, by taking this into account, for the upper and lower headlands, respectively, the length of the headland path that connects the two lanes  $h$  and  $k$ , a better approximation is given by the following expressions (Bochtis *et al.*, 2009):

$$|k-h|_u = \sum_{q=\min(h,k)}^{q=\max(h,k)-1} \Delta(V_{q+1}^u, V_q^u) \quad [3]$$

or

$$|k-h|_d = \sum_{q=\min(h,k)}^{q=\max(h,k)-1} \Delta(V_{q+1}^d, V_q^d)$$

Let  $X_{min} : K \rightarrow \mathbb{R}$  returns the theoretical turning distance of the  $\Pi$ -turn and  $T$ -turn,  $R_{min} : K \rightarrow \mathbb{R}$  returns the theoretical turning distance of the  $R$ -turn. We defi-

ne the variable which is  $\gamma = 0$  if the AU executes an  $R$ -turn, otherwise  $\gamma = 1$ . Then, the turning distance  $H_{min}$  can be expressed as:

$$H_{min} = X_{min} \cdot \gamma + R_{min} \cdot (1 - \gamma) \quad [4]$$

Regarding the non-working distances that the AU has to travel between patches within field can be given by the operator  $\Delta: \mathbb{R}^2 \times \mathbb{R}^2 \rightarrow \mathbb{R}$  for given the vectors of their coordinates, three types of which are the following:

(a)  $\Delta(V_k^u, V_k^u) t \in T$ , returns a distance between the point  $t$  and the upper ending of lane  $k$ .

(b)  $\Delta(V_k^l, V_k^d) t \in T$ , returns a distance between the point  $t$  and the lower ending of lane  $k$ .

(c)  $\Delta(V_k^l, V_k^e) t, e \in T$ , returns a distance between the two points  $t$  and  $e$  in the lane  $k$ .

Thus, using the previous formulation and the definitions of the functions including  $p^{-1}(\cdot)$ ,  $f_1(\cdot)$  and  $f_2(\cdot)$  it easily arises that the total non-effective distances of the two patches covered consecutively by the agricultural robotics at the steps  $s$ , and  $s + 1$ , can be given by:

(1) if  $f_1[p^{-1}(s)] \neq f_1[p^{-1}(s+1)]$ , and  $p^{-1}(s+1) \in T_1$ , then

$$d_{ij} = \Delta(V_{f_1[p^{-1}(s)]}^u, V_{f_1[p^{-1}(s)]}^{p^{-1}(s)}) + H_{min} + \Delta(V_{f_1[p^{-1}(s+1)]}^u, V_{f_1[p^{-1}(s+1)]}^{p^{-1}(s+1)}) \quad [5]$$

(2) if  $f_1[p^{-1}(s)] \neq f_1[p^{-1}(s+1)]$ , and  $p^{-1}(s+1) \in T_2$ , then

$$d_{ij} = \Delta(V_{f_1[p^{-1}(s)]}^{p^{-1}(s)}, V_{f_1[p^{-1}(s)]}^d) + H_{min} + \Delta(V_{f_1[p^{-1}(s+1)]}^{p^{-1}(s+1)}, V_{f_1[p^{-1}(s+1)]}^d) \quad [6]$$

(3) if  $f_1[p^{-1}(s)] = f_1[p^{-1}(s+1)]$ , and  $f_2[p^{-1}(s)] \neq f_2[p^{-1}(s+1)]$ , then

$$d_{ij} = \Delta(V_{f_1[p^{-1}(s+1)]}^{p^{-1}(s+1)}, V_{f_1[p^{-1}(s)]}^{p^{-1}(s)}) \quad [7]$$

(4) if  $f_1[p^{-1}(s)] = f_1[p^{-1}(s+1)]$ , and  $f_2[p^{-1}(s)] = f_2[p^{-1}(s+1)]$ , then

$$d_{ij} = 0 \quad [8]$$

And the total non-working distance travelled by agricultural robot for covering each patch in the whole field is given by:

$$J(\mu) = \sum_{s=1}^{|T|} d_{p^{-1}(s)p^{-1}(s+1)} \quad [9]$$

### Mixed networking graph

The problem that we are focused on is the optimization of field efficiency by minimizing the distance travelled  $J(\mu)$  during the spatially selective operations within cropping field. Due to the total non-working distance occurs both at the headlands and inside the field interior. Hence, the traversal of each patch in the whole field can be represented as the traversal of a mixed graph  $G = (T, E, A)$ , which consists of a finite set

of graph nodes  $T = \{1, 2, 3, \dots, |T|\}$ , a set of edges  $E = \{(i, j) | i, j \in T\}$ , and a set of arcs  $A = T \times T$ . Each node in the graph corresponds to a single ending of the patch in the lane and the number of nodes is twice as much as the number of patches. Traversing all field patches is equivalent to visiting all nodes in  $G$ . Each edge  $E_{ij}$  or arc  $A_{ij}$ , ( $i \neq j$ ), joins node  $i$  to node  $j$ , in this sequence. Each edge  $E_{ij}$  or arc  $A_{ij}$  is associated with a cost  $d_{ij}$  which corresponds to the non-working distance that the AU moves from patch  $i$  to patch  $j$ . We take lane 1 and lane 2 in Fig. 1 for a simple example to illustrate this representation, in which the lane 1 has one patch and lane 2 has two patches. The spatially selective operations to cover these three patches are represented as a mixed graph in Fig. 2, where  $d_{ij}^u$  and  $d_{ij}^d$  denote the travelled distances from node  $i$  to node  $j$  via the upper and lower headlands, respectively. The dotted lines denote the connectivity between the upper and lower endings of three patches, in which the weight  $d_{ij} = 0$  denotes the travel of the AU from node  $i$  to node  $j$  without non-productive distance.

In accordance with practice that the machine has to complete the route ‘‘barn-field-barn’’, we assume that the AU has an initial location before the operation starts, which is its current physical position, and that the AU will back to the initial location after the operation has been completed. Therefore, the total operation of covering all patches in a given field must be represented by an extended graph  $G'$  which contains the patch coverage graph  $G$ , plus an initial location node 0 (e.g., a barn); letting  $N = \{0\} \cup T$  be the set of the nodes of the new graph. The cost of connecting node 0 to any other node  $j$  is equal to the distance  $d_{0j}$ , so that the AU has to move from its initial position to reach the node  $j$  of lane  $k$ , which is given by:

$$d_{0j} = \begin{cases} u_{0k} + \Delta(V_k^u, V_k^u), j \in T_1 \\ d_{0k} + \Delta(V_k^l, V_k^d), j \in T_2 \end{cases} \quad [10]$$

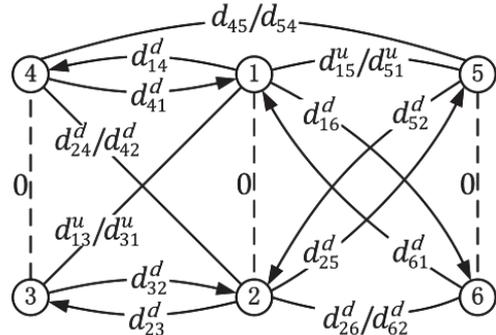


Figure 2. A mixed graph of patches coverage traversal.

where  $u_{0k}$  and  $d_{0k}$  denote the distance from its initial position to the upper and lower endings of lane  $k$ , respectively,  $\Delta(V_k^u, V_k^l)$  and  $\Delta(V_k^l, V_k^d)$  return the distance from the upper and lower endings of lane  $k$  to node  $j$ .

Regarding the cost that the AU has to move from any node  $j$  to node 0, it is equivalent to the travelled distance  $d_{j0}$ . Consider the route “barn-field-barn” completed by the AU, in this work we assume that the AU has to exit the field from as close as possible to the barn or the initial location. Consequently the cost is given by:

$$d_{j0} = \min\{\Delta(V_k^u, V_k^l) + u_{0k}, \Delta(V_k^l, V_k^d) + d_{0k}\}, j \in T \quad [11]$$

### Optimization of field efficiency

Field efficiency is defined as the ratio between the effective travelling distance (the distance that the AU travels while operating) and the total distance travelled by the AU. Due to the effective travelling distance for covering all patches is constant, the improvement of field efficiency depends on the minimization of non-working distance. Summing up all mentioned above, for any traversal sequence  $\mu = \langle p^{-1}(1), p^{-1}(2), \dots, p^{-1}(|T|) \rangle$ , the total non-working distance includes the total non-working distance for covering all patches as well as the distances from the initial position to the first node and from the last node to the initial position. The sum of these non-working distances is expressed as:

$$D(\mu) = d_{0p^{-1}(1)} + J(\mu) + d_{p^{-1}(|T|)0} \quad [12]$$

The optimal traversal sequence for covering all patches which maximizes field efficiency is the permutation  $\mu^*$  which constitutes the solution of the following optimization problem (Bochtis & Vougioukas, 2008):

$$\mu^* = \arg \min D(\mu) \quad [13]$$

Solving the above problem is equivalent to solving the Asymmetric TSP (ATSP) based on the extended graph  $G'$ . The ATSP is a NP-complete combinatorial problem. Various heuristic algorithms have been developed to find near-optimal (and sometimes optimal) solutions, such as branch-and-bound (Turkenteen *et al.*, 2008), ant colony algorithm (Gambardella & Dorigo, 1996), and genetic algorithm (Nagata & Soler, 2012).

In this work, an improved genetic algorithm, *i.e.*, partheno-genetic algorithm proposed by Li & Tong (1999) was employed. The traversal sequence for covering all patches was encoded as an ordinal string which represents a chromosome. In a chromosome, each gene represents a node where the first gene and the last gene remain fixed. They are respectively equal to the initial location and the final location of the AU. The flowchart of the optimization of field efficiency based on partheno-genetic algorithm is shown in Fig. 3, and a simple description is following:

(i) Determine the prior information, including field dimensions, operating width and the location information of patches. Data may be retrieved from a

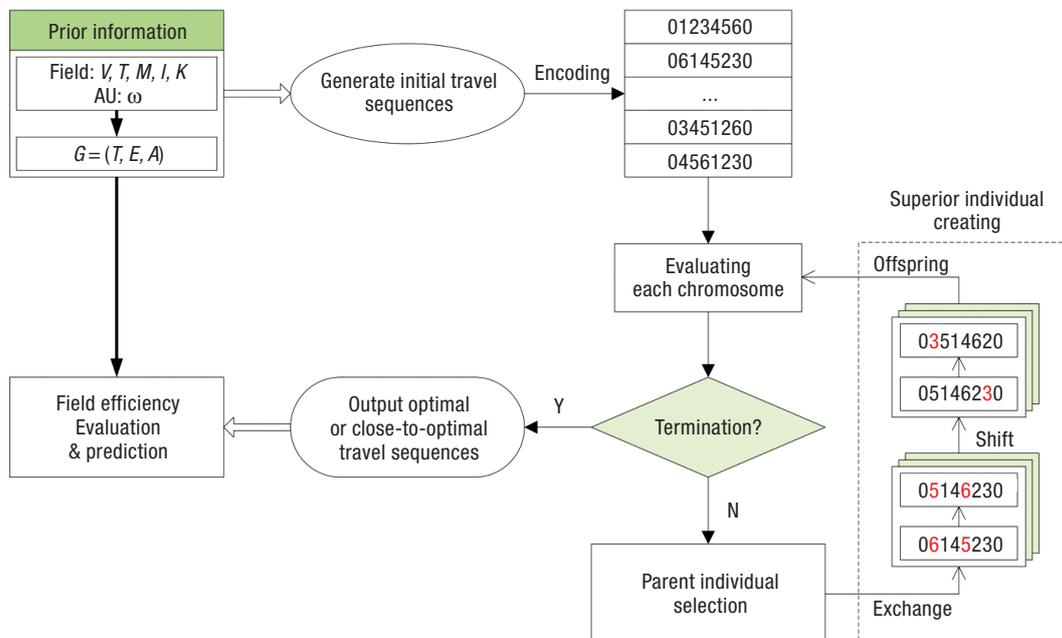


Figure 3. Flowchart of the optimization of field efficiency based on partheno-genetic algorithm.

Geographic Information System (GIS), machinery and operations databases like operations logs, aerial photos, and yield maps.

(ii) Based on this information, represent the data as a connected graph  $G' = (T, E, A)$  with weights  $d_{ij}$ .

(iii) Generate optimal patch sequence by solving the Asymmetric TSP (ATSP) based on the flowchart of partheno-genetic algorithms.

(iv) Compute the theoretical distance travelled based on the optimal patch sequence and estimate the field efficiency.

## Results

An example is presented in order to demonstrate the optimal paths that result from the methodology mentioned above. A cropping field ( $44^{\circ} 20' 31''$  N,  $85^{\circ} 45' 23''$  E) from Northwestern China was selected, the area of which was approximately 16.25 ha ( $422 \text{ m} \times 385 \text{ m}$ ). The operating width of the agricultural robot used was measured as 21 m, and the minimum turning radius was 4 m. To implement the model, the practical planning was done using the technical programming language.

The planning was initiated by acquiring and loading a priori information in terms of an aerial map into Matlab identifying the field and associated boundaries in terms of poor patches. According to the operating width of the agricultural robot, the field area with poor patches was divided into seven lanes, each lane included one or two sub-patches, and a unique number for each



Figure 4. Prior information and coordinate system.



Figure 5. Optimal robot paths for covering nine patches.

sub-patch was given as shown in Fig. 4. The results were transferred to a structure that was used to generate the mixed graph  $G = (T, E, A)$ . Cost was assigned to all edges or arcs based on the notion of distance.

Next, the ATSP was solved based on the mixed graph. The optimal sequence for covering nine patches in the given field was computed by the optimization procedure and was found to be  $\mu^*_1 = \langle 9, 8, 7, 6, 5, 4, 3, 2, 1 \rangle$ , and the total non-working distance for this sequence was 1,704.8 m. The travel trajectory for this field operation is as shown in Fig. 5.

In order to validate the potential benefit of the proposed approach in this work, we compared the above-mentioned results to those obtained by using the model of Bochtis & Vougioukas (2008). The same work conditions were input in the model, and the optimal sequence was found to be  $\mu^*_2 = \langle 9, 8, 7, 6, 5, 4, 3, 2, 1 \rangle$ , consequently the total non-working distance for this sequence was 1,989.4 m. The travel trajectory of this sequence is as shown in Fig. 6.

Table 1 presents the data from the two methods. The field efficiency (distance based) is 34.6% for proposed method in this work and 31.2% for Bochtis's method, and the non-working distance for the proposed method is reduced by 284.6 m. The non-working distance inside the field interior contributes to 57.8% (by proposed method) and 62.1% (by Bochtis's method) of the total travelled distance, while the headland turnings, according to the computed data contribute to 7.6% and 6.8% of the total travelled distance.

For each lane in the given field, the savings on the non-working distance travelled by the AU and the size



**Figure 6.** Robot paths for covering nine patches with Bochtis's method.

and location of patches are shown in Table 2. Table 2 shows that the savings on the non-working distance are zero for lanes 4 to 7, respectively. It denotes the exiting of the AU by reverse driving is not beneficial to reduce the non-working distance because of the location of patches is close to the middle endings of their lanes. In contrast, by doing so, the non-working distance can be reduced for the case of lanes 1 to 3.

## Discussion

An algorithmic approach for computing traversal sequences for an agricultural robot that has to treat some poor-patches in a field with row crops has been developed. The traversal of patches in the field was expressed as the traversal of a mixed weighted graph, and

**Table 1.** Comparison between the data from the proposed methods in this work and Bochtis's

Comparison items	Proposed method	Bochtis's method
Total travelled distance (m)	2,605.8	2,890.4
Effective distance (m)	901.0	901.0
Total non-working distance (m)	1,704.8	1,989.4
Savings in total non-working distance (%)	—	14.3
Turning distance (m)	198.8	195.4
Field efficiency (distance) (%)	34.6	31.2

then the problem of finding an optimal traversal sequence was formulated as an asymmetric traveling salesman problem and solved by the partheno-genetic algorithm. The research results showed that the non-working distance can be reduced by using algorithmically computed optimal patch sequences. In comparison to the existing methods, such as the method of Bochtis & Vougioukas (2008), which computes the optimal lane sequences towards minimizing the total turning distance, but the proposed method in this work has more potential to minimize the total non-working distance travelled inside the field interior during spatially selective operations. This makes sense, since the non-working distances inside the field interior contribute about 60% of the total travelled distance (Table 1). Furthermore, this fact was clearly demonstrated at the concrete case mentioned above, where the savings on the non-working distance could be as high as 14.3% in contrast to the results from the method of Bochtis & Vougioukas (2008) and all that derived from inside the field interior. This implies that the method of Bochtis & Vougioukas (2008) is only applicable to minimizing the non-working distance travelled during headland turnings.

**Table 2.** The savings of the non-working travelled distance and the size and location of patches in each lane

Lanes	Patch proportion in its lane (%)	Patch location in its lane	Saving in non-working distance (m)
1	12.2	Lower	154
2	36.9	Lower	83
3	36.1	Lower	51
4	44.7	Lower or mid	0
5	47.8	Upper or mid	0
6	41.6	Upper or mid	0
7	14.8	Mid	0

Besides the computed optimal sequences for covering each patch in the field, the introduction of two-way robot is also important to minimize the non-working distance inside the field interior. There were cases during spatially selective operations that the AU entered a new lane to treat one and only small patch in the lane by moving from headland to headland. Consequently, the AU drove for a long non-working distance on the current lane. In order to avoid this, the AU can move backward to exit the lane from the entry-point after completing its operation, for this case such as the lane 1, lane 2 and lane 3 in Fig. 5.

Regarding the influence of the size and location of patches in the field with respect to the savings on the non-working distance, as it was expected there were several cases to be discussed. The Table 2 shows that three types of patch location including the upper, middle and lower area of the lane have influences on the field efficiency:

— The middle area of lane: if a poor-patch is located in the middle area of its lane, for covering this patch, the AU has to travel across the middle area of the lane. It implies that the exiting of the AU by reverse driving is not beneficial to reduce non-productive distance and the savings on non-working distance are zero. The patch 9 in Fig. 5 is a case in point.

— The upper or lower area of lane: if a poor-patch is close to the upper or lower headland of its lane, while the length of the patch is less than a half of the length of its lane, the reverse driving of the AU for exiting is beneficial to reduce non-productive distance, otherwise, that is not. As illustrated in Fig. 5, the patch 3 was close to the lower headland and its length was less than a half of lane 3. Consequently, the reverse driving of the AU for exiting the lane 3 after completing its operations was dedicated to improving field efficiency (Table 2).

— The upper and lower area of lane: in a lane, if some poor-patches are located in the upper area and others in the lower, respectively, while the total length of all patches are less than a half of the length of the whole lane, the reverse driving of the AU for exiting is beneficial to reduce non-working distance, otherwise not.

Therefore, the savings on the non-working distance depend on the length proportion and location of the patch in its lane. The proposed method proved to be effective to generate optimal patch sequences for the minimization of the non-working distance, but this work does not take into account the capacity of the AU tank. This can be considered in future research.

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